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ESSAY ON GRAVITATION¹**The Holographic Interpretation of Hawking Radiation**ALESSANDRO FABBRI²*Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, C.**Dr. Moliner 50, Burjassot-46100, Valencia, Spain.*GIOVANNI PAOLO PROCOPIO³*D.A.M.T.P., Centre for Mathematical Sciences, University of Cambridge,**Wilberforce Road, Cambridge CB3 0WA, U.K.***Abstract**

Holography gives us a tool to view the Hawking effect from a new, classical perspective. In the context of Randall-Sundrum braneworld models, we show that the basic features of four-dimensional evaporating solutions are nicely translated into classical five-dimensional language. This includes the dual bulk description of particles tunneling through the horizon.

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The problem of understanding why and how black holes emit particles is not an easy one. It is intriguing that the motivation that led to the discovery of Hawking radiation came from classical considerations, namely the analogy between the laws of black hole mechanics and the laws of thermodynamics. However, already on dimensional grounds one realizes that in order to construct a quantity with the dimensions of an entropy starting from the area of the event horizon A_H , quantum mechanics needs to enter into the game. Indeed, the only possibility is $S \sim k_B A_H / A_{\text{Pl}}$, where A_{Pl} is Planck area and k_B Boltzmann's constant. This, in turn, implies that the black hole temperature T is $O(\hbar)$ and, correctly, vanishes in the classical limit.

The actual derivation of particles emission by black holes [1] requires the quantization of matter fields in the dynamical background describing the formation of a black hole via gravitational collapse. This is the crucial feature allowing the initial vacuum state of our radiation fields to be detected, by asymptotic observers in the future asymptotic region at late times, as a mixed thermal state at the Hawking temperature (for a Schwarzschild black hole of mass M and with $c = G = 1$)

$$T_H = \frac{\hbar}{8\pi M k_B} . \quad (1)$$

A visual mechanism useful to understand the reason for the quantum instability of the Schwarzschild spacetime was first proposed in [2]. Creation of particle-antiparticle virtual pairs in the near horizon region may be such that the negative-energy antiparticle is absorbed by the black hole, leaving its positive-energy partner free to escape to infinity. Estimates for this process indicate that

the emitted particle has the correct energy-temperature dependence we expect for thermal radiation, i.e. $E \sim k_{\text{B}}T_{\text{H}}$.

An alternative way to see this process is that of a particle tunneling through the horizon. A recent derivation of the Hawking effect using this idea was performed in [3], where it was noticed that in order for the particle to tunnel through a ‘real barrier’ energy conservation must be enforced. Indeed, once a particle of energy E escapes from the black hole and reaches infinity the black hole must shrink in size from $2M$ to $2(M - E)$. Computation of the emission rate at leading order in the particle energy E gives exactly the Boltzmann factor for thermal radiation at the Hawking temperature T_{H} .

The idea of energy conservation takes us back to the real time dependent gravitational collapse problem. Hawking’s derivation, in fixed background approximation, implies that the black hole radiates with luminosity

$$L \sim \frac{(k_{\text{B}}T_{\text{H}})^2}{\hbar} \sim \frac{\hbar}{M^2} \quad (2)$$

for an infinite amount of time. This is at odds with the fact that the black hole has a finite mass M . The classical background spacetime itself has to be modified by the quantum corrections and in particular the black hole mass will reduce at a rate given by L . In the near-horizon region, a good approximation to the evaporating solution is given by the advanced Vaidya metric [4]

$$ds^2 = - \left(1 - \frac{2M(v)}{r} \right) dv^2 + 2drdv + r^2 d\Omega_2^2, \quad (3)$$

where the apparent horizon $r_{\text{AH}} = 2M(v)$ recedes according to

$$\frac{dr_{\text{AH}}}{dv} = -2L. \quad (4)$$

Because of the evaporation apparent and event horizons, coincident for the static Schwarzschild solution, separate. In the large mass limit one can estimate the location of the event horizon by integrating (4) over the typical timescale of the process $\Delta v \sim M$ to get

$$\frac{r_{\text{EH}} - 2M}{2M} \sim -L . \quad (5)$$

Therefore, a new region ($r_{\text{EH}} < r < r_{\text{AH}}$), called in [5] the “quantum ergosphere” and absent in the classical solution, forms. In this region photons are locally trapped but being outside the event horizon they can cross the apparent horizon at a later time and propagate to infinity. One can then view the (classically forbidden) tunneling trajectory of the particle across the horizon as the (physically allowed) trajectory across the apparent horizon in a quantum corrected evaporating spacetime.

We propose here a new interpretation of the Hawking effect which is entirely classical and which, nevertheless, is able to capture all the features here described. We shall use holography, in particular the predicted duality, in Randall-Sundrum braneworld models, between a classical theory in AdS_5 (the bulk) and a theory living on our 4D universe (the brane) where classical gravity is coupled to quantum matter fields. At linear level, the zero mode of the 5D gravitons, bound to the brane, reproduces 4D Newtonian gravity at large distances, whereas massive modes, propagating in the bulk, induce the quantum effects on the brane and thus are dual to the quantum matter fields [6].

Application of these ideas beyond the linear level has led to the conjecture [7] that for large masses “4D black holes localized on the brane found by solv-

ing 5D Einstein equations in AdS_5 are quantum corrected black holes and not classical ones". Basing our arguments on this conjecture we can then describe the Hawking effect from a new, classical prospective.

The braneworld analog of the Schwarzschild black hole is the Randall-Sundrum Schwarzschild black string

$$ds^2 = e^{-2k|z|} \left[- \left(1 - \frac{2M}{r} \right) dv^2 + 2dvdr + r^2 d\Omega^2 \right] + dz^2, \quad (6)$$

where k is the inverse of the AdS_5 length. This metric gives the classical Schwarzschild spacetime on the brane ($z = 0$), but it is problematic for several reasons. This solution is singular at the AdS horizon $z = \infty$ and, moreover, it is unstable to linear perturbations (the Gregory-Laflamme instability [8]), implying a time dependent evolution away from (6). If the holographic conjecture is correct, the actual time-dependent solution will reproduce a quantum corrected evaporating black hole on the brane. Unfortunately such solution is not known. However, as suggested from the linear analysis, the quantum effects on the brane are classically given by those gravitational waves (bulk Kaluza-Klein massive modes) which propagate along null geodesics in the bulk.

In braneworld there are two different definitions of horizons: the brane apparent horizon $r_{\text{brane}}^{\text{AH}}$, defined with respect to photons moving along null geodesics on the brane, and the bulk apparent horizon $r_{\text{bulk}}^{\text{AH}}$, defined with respect to gravitons that propagate in the full 5D spacetime. In the case of (6) we have two different families of null geodesics [9], those restricted to $z = \text{constant}$ (in particular on the brane $z = 0$) and those propagating nontrivially in the

bulk satisfying

$$\dot{z} = -\frac{1}{k\lambda} \quad (7)$$

where λ is the affine parameter. Intuitively, one would think that the first family of geodesics at $z = 0$ defines the brane apparent horizon, while the second one (7) gives the bulk apparent horizon. However, this is not so.

The first family ($z = \text{constant}$), equivalent to the usual Schwarzschild null geodesics, defines the horizon at $r = 2M$ where $dr/dv = 0$. For the second family (7) we find that $r = 2M$ is not a solution and, also, that in the near horizon region [10]

$$\frac{dr_{\text{bulk}}}{dv} \sim -\frac{1}{(4Mk)^2} e^{2kz}, \quad r_{\text{bulk}}(z) \sim 2M \left[1 - \frac{e^{2kz}}{(4Mk)^2} \right]. \quad (8)$$

If we now project (8) to the brane ($z = 0$) and use the holographic relation $\frac{1}{k^2} \sim \hbar N^2$, where N^2 corresponds to the (huge) number of degrees of freedom in the dual theory, we get

$$\left. \frac{dr_{\text{bulk}}}{dv} \right|_0 \sim -\frac{\hbar N^2}{M^2} \equiv -2L, \quad \frac{r_{\text{bulk}}(0) - 2M}{2M} \sim -L. \quad (9)$$

The similarity with (4) and (5) is surprising. What is the physical meaning of the surface $r_{\text{bulk}}(z)$? We cannot identify it with the bulk apparent horizon, as for the black string $r_{\text{brane}}^{\text{AH}} = r_{\text{bulk}}^{\text{AH}} = 2M$.

Note, however, that the discovery of the Hawking effect was performed in fixed background approximation and that it implied that the Schwarzschild solution is quantum mechanically unstable and must be modified by the quantum corrections. In a similar fashion, if the holographic interpretation (9) is correct

it is reasonable to expect that bulk massive modes, traveling along the nontrivial bulk geodesics (7) and responsible for the quantum effects on the brane, will modify the black string to a time dependent solution with the horizon satisfying the dual behavior (9) on the brane.

Following this line of reasoning, the surface $r_{\text{bulk}}(z)$ will become the bulk apparent horizon $r_{\text{bulk}}^{\text{AH}}(z)$ of such time dependent solution. This is somehow confirmed by the fact that, as shown in [11], in time dependent braneworld black hole solutions brane and bulk apparent horizons are generically distinct and in particular $r_{\text{bulk}}^{\text{AH}} < r_{\text{brane}}^{\text{AH}}$. Moreover, according to (9) the projection on the brane of $r_{\text{bulk}}(z)$ will play the role of the event horizon in the dual evaporating theory. Gravitons can be emitted from the region $r > r_{\text{bulk}}^{\text{AH}}$ on the brane to the bulk, and in particular from the (large) quantum ergosphere $r_{\text{bulk}}^{\text{AH}} < r < r_{\text{brane}}^{\text{AH}}$ (large because its size is proportional to N^2). This is indeed the region from where we expect the bulk massive modes dual to the Hawking quanta to be emitted.

On the base of this interesting analogy between classical bulk and quantum brane effects we propose a mechanism that represents the dual bulk description of particles tunneling through the horizon. This is represented in fig. 1, in which bulk massive modes leaving the brane to the bulk bounce back to the brane just outside the brane apparent horizon. Whether this process really takes place or not in the full time-dependent 5D solution could then tell us if tunneling is the actual process behind the Hawking effect.

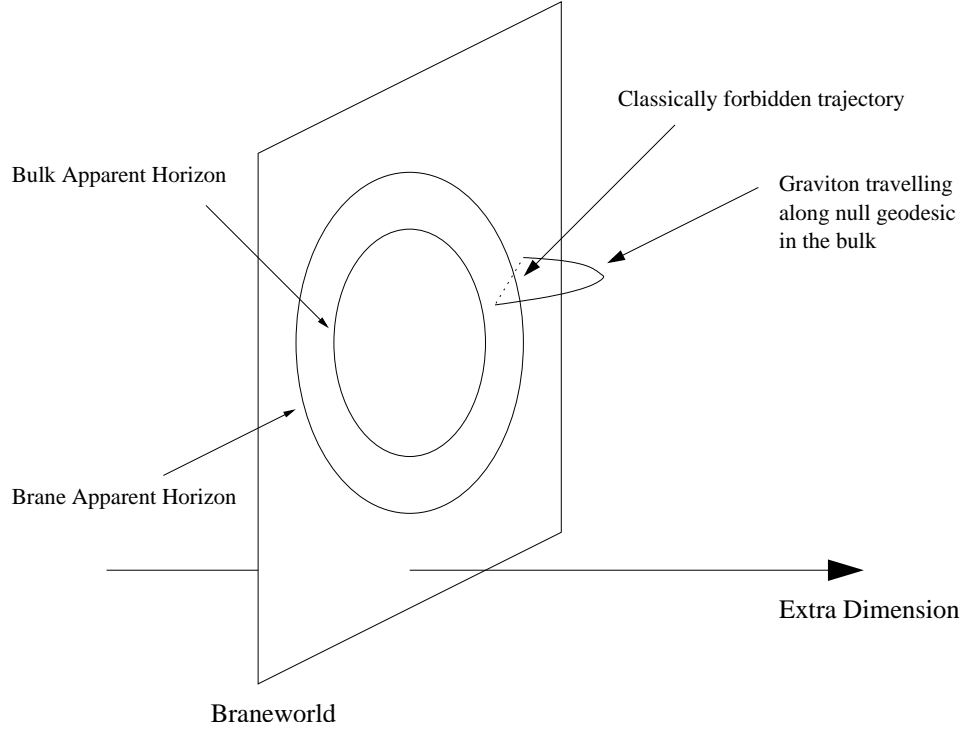


Figure 1: Classical bulk dual of particles tunneling through the horizon.

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